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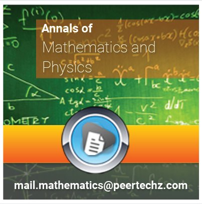

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Research Article

Black quanta. On the thermodynamics of the black holes

Abstract

It is shown that the quantized internal motion of the black holes consists of Planck quanta (Planck mass, length, time, etc), which may be called black quanta. The mass of the black hole is a integral multiple of the Planck mass, and the radius of the black hole (Schwarzschild radius) is an integral multiple of the Planck length. This circumstance arises from the proportionality of the black hole radius and mass. The statistical physics and the thermodynamics of the black holes are derived herein from the statistical motion of the black quanta.

It is well known that bodies may suffer a gravitational collapse, providing their mass is sufficiently large, their dimensions are sufficiently small and their measurable internal motion ceased [1,2]. In such a state they are black holes. We cannot have any information about their internal state. Any mass or radiation signal falls in the infinite space-time singularity of the black holes. In order to get a qualitative criterion of the black-hole condition we use GM^2/R for the gravitational energy of a spherical mass M with radius R, where $G \simeq 6.7\times 10^{-8} cm^3/g\cdot s$ is the gravitational constant; if $GM^2/R > Mc^2$ (where $c \simeq 3\times 10^{10} cm/s$ is the speed of light in vacuum), *i.e.* if $M/R > c^2/G$, the mass collapses; the condition may also be written as $R < R_h$, where $R_h = GM/c^2$ is close to the Schwarzschild radius $r_h = 2R_h$. We take $R_h = GM/c^2$ as the radius of any black hole with mass M.

The radiation inside a black hole (of any kind), being delocalized, moves in the highly curved space-time of the black hole. Consequently, there appear quantum-mechanical transitions [3] and, near the black-hole horizon (radius), radiation quanta may escape, for a while, from the black hole. This is related to the so-called Hawking fluctuating radiation [4]. If the black hole fluctuates, *i.e.* if its mass M and radius R_h fluctuate, we may think that the black hole has an internal statistical motion and a thermodynamics. The current description of the statistical motion and the thermodynamics of a black hole raises serious questions [2]. This description assumes usually that the area of the black hole can only increase [5], as a consequence of accretion, so this area divided by the Planck area is a dimensionelss parameter which can only increase. Therefore, the reasoning goes further, it is an entropy, which may be equalized with the ratio of any two energies, one being viewed as heat, the other as temperature. This argumentation is insufficient to admit that the area of the black holes is proportional to their entropy.

The way to the statistical physics and the thermodynamics of a black hole is provided by the quantum motion of the radiation inside the hole. Indeed, the space quantization requires

$$R_h = n\lambda\ , \tag{1}$$

where n is any positive integer and λ is the radiation wavelength; on the other side, the time quantization requires

$$\varepsilon = Mc^2 = n'\frac{hc}{\lambda} \tag{2}$$

for the energy ε of the black hole, where h is Planck's constant and n' is a positive integer. Inserting equation (1) in equation (2) we get

$$\varepsilon = Mc^2 = n'\frac{hc}{\lambda} = nn'\frac{hc}{R_h} = nn'\frac{hc^3}{GM}\ , \tag{3}$$

whence we get

$$M = \sqrt{nn'}\sqrt{\frac{hc}{G}} = \frac{c^2}{G}R_h = n\frac{c^2}{G}\lambda\ ; \tag{4}$$

obviously, these equations can only be satisfied if $n = n'$ and

$$M = n\mu\ ,\ \mu = \sqrt{\frac{hc}{G}}\ , \tag{5}$$

$$R_h = n\lambda\ ,\ \lambda = \sqrt{\frac{hG}{c^3}}.$$

We can see that the mass is an integral multiple of the Planck mass μ and the radius R_h is an integral multiple of

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the Planck length λ (up to a $\sqrt{2\pi}$ factor). The energy of a black hole $\mathcal{E} = n\varepsilon_0$ is an integral multiple of the fundamental Planck energy $\varepsilon_0 = \mu c^2 = \sqrt{\frac{hc^5}{G}}$ (Planck "temperature"), which corresponds to the Planck wavelength $\lambda = hc / \varepsilon_0$. This particular circumstance of the quantization of the motion arises from the black hole condition $R_h = GM / c^2$.

The frequency $\nu = \varepsilon_0 / h = \sqrt{\frac{c^5}{hG}} \simeq 10^{43} s^{-1}$ (the reciprocal of the Planck time) is very high; The Planck energy is of the order $\varepsilon_0 \simeq 6 \times 10^{16} erg$, or $\simeq 10^{28} eV$, or $\simeq 10^{32} K$. Because of the extremely high frequency of these quanta we may call them black quanta.

The statistical physics of the quantum-mechanical motion described above is immediate; it corresponds to a single quantum-mechanical state of energy ε_0 occupied by n black quanta; the number n is the (main) statistical variable. The thermodynamic potential Ω (free energy F) is

$$\Omega = F = -T \ln \sum_n e^{-\beta\varepsilon_0 n} = T \ln\left(1 - e^{-\beta\varepsilon_0}\right) , \qquad (6)$$

where $T = 1/\beta$ is the temperature (the chemical potential is zero and we include $n = 0$ in summation); the mean occupation number is

$$\bar{n} = \frac{1}{e^{\beta\varepsilon_0} - 1} , \qquad (7)$$

the mean energy is $E = \bar{\mathcal{E}} = \bar{M}c^2 = \bar{n}\varepsilon_0$ and the entropy is

$$S = -\frac{\partial\Omega}{\partial T} = \frac{\beta\varepsilon_0}{e^{\beta\varepsilon_0} - 1} - \ln\left(1 - e^{-\beta\varepsilon_0}\right) ; \qquad (8)$$

we can see that the black hole has a mean mass $\bar{M} = E / c^2$ and a mean radius $\bar{R}_h = G\bar{M} / c^2$. Also, we can see that $S \to 0$ for $T \to 0$ (according to the third principle of thermodynamics). The relative fluctuation in the occupation number is $\delta n / \bar{n} = \sqrt{e^{\beta\varepsilon_0} + 1} - 1$. This is also the relative fluctuation in energy, mass, radius; it can be related to the entropy fluctuation by using $S = (\bar{n}+1)\ln(\bar{n}+1) - \bar{n}\ln\bar{n}$ (which follows from equations (7) and (8). This latter formula shows that at equilibrium, for a given mean energy, the maximum of the entropy gives the mean occupation number $\bar{n}$ in equation (7); and any change out of equilibrium decreases the entropy, as expected (in agreement with the law of increase of entropy - the second principle of thermodynamics).

At eqilibrium the thermodynamic potential is stationary ($d\Omega = 0$); since, on one hand, in equilibrium transformations, $d\Omega = -SdT$ and, on the other, $d\Omega = dF = d(E - TS)$, we get $dE = TdS$, *i.e.* the change in energy is, in fact, a change in the amount of heat. The temperature T is a measure of the internal energy E of the black hole (proportional to the mean mass $\bar{M}$, or mean size $\bar{R}_h$); according to equation (7), for $T \ll \varepsilon_0$ no Planck mass (black quanta) is excited inside the black hole and $\bar{M} \simeq (\varepsilon_0 / c^2) e^{-\varepsilon_0/T} \ll \mu$; on the contrary, for $T \gg \varepsilon_0$ a large number of black quanta are excited and $\bar{M} = (c^2 / G)\bar{R}_h \simeq T / c^2 \gg \mu$.

The black holes may have an electric charge and, also, they may rotate; these external parameters may be included in the thermodynamics by adding to the energy $\mathcal{E} = n\varepsilon_0$ the energies $q\Phi$ and $-\mathbf{\Omega L}$, where q is the electric charge, Φ is its electric potential, $\mathbf{\Omega}$ is the angular frequency and $\mathbf{L}$ is the angular momentum; since $\Phi \sim -1 / R_h$, the charge contribution is proportional to $-1/n$; similarly, since $L \sim R_h^2$, the rotational term brings a contribution proportional to $-n^2$. The full expression of the total energy, which is a function of n, is introduced in equation (6), which gives the thermodynamic potential. The contributions brought by electric charges or rotations to the thermodynamic properties are very small, such that they may be treated as corrections to the potential given by equation (6).

In conclusion, we may say that the quantization of the motion inside a black hole identifies elementary excitations called black quanta, which correspond to Planck mass, wavelength, energy, frequency, time, etc. The statistics and the thermodynamics of these elementary excitations are computed explicitly in this paper.

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