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Review Article

Squaring the Circle Using Modified Tartaglia Method

Abstract

The paper presents a modified Tartaglia method. Tartaglia proposed a simple approach to perform an approximate quadrature of the circle. His construction results with the number $\pi \approx 3.125$. Using a similar construction as Tartaglia but with different proportions improves the accuracy of his method.

Introduction

Italian mathematician Niccolo Fontana Tartaglia (1500–1557) in his book [1], presented the following approximate squaring the circle. He started from a given square with diagonal. He transformed the square into the circle dividing its diagonal into ten equal parts; the corresponding circle as its diameter (d) has eight parts of the diagonal of the square. It can be seen as Tartaglia used a set square with the sides in proportion 8:10 or 4:5. This idea (using a set square) will be further developed in this short note.

Consider the square with the side of length a. Its diagonal is $z = a\sqrt{2}$. Tartaglia method gives the following relation $\pi r^2 = \pi \left[\left(\frac{d}{2} \right)^2 \right] = \pi \left[\left(\frac{0.8z}{2} \right)^2 \right] = \pi \left[\left(\frac{0.8a\sqrt{2}}{2} \right)^2 \right] = a^2$. This condition that the areas of the square and the circle constructed in such way are equal results in the number $\pi \approx 3 \frac{1}{8}$. Thus the approximation is the same as was obtained in ancient Babylon.

Method

The following question holds: is it possible to improve this method? The answer to this question is yes, it can be done better. Now the idea is that rather to scale the diagonal z by 0.8, lets try to find the scale factor x, which gives better approximation. For this purpose consider more general approach based on the following relation $\pi r^2 = \pi \left(\frac{d}{2} \right)^2 = \pi \left(\frac{xz}{2} \right)^2 = \pi \left(\frac{x a\sqrt{2}}{2} \right)^2 = a^2$. From this equation the factor x is easily determined as $x = \sqrt{2/\pi} \approx 0.797884560802865\dots$. Using the continued fraction, we have the following 19 first terms for $x = [0; 1, 3, 1, 18, 9, 4, 1, 4, 3, 2, 1, 3, 1, 1, 2, 1, 5, 1, \dots]$. The infinite continued fraction representation for x is very useful here. Using rational approximations to the number x it is possible to construct a set square which can be used to perform the approximate quadrature of the circle.

Results

The corresponding convergents of the determined continued fraction for x are presented in Table 1. The table also contains the approximate values for the number pi. By constructing the set square with sides in the proportion 35567099:44576748 the tool will do approximate quadrature of the circle with $\pi \approx 3.141592653589795$ where true $\pi \approx 3.1415926535897931$.

Table 1. Numerator, denominator, fraction and the approximation to the number pi.

Table 1: Numerator, denominator, fraction and the approximation to the number pi.

Term	Numerator	Denominator	Approximation to x	Approximation to pi
1	0	1	0.0	*
2	1	1	1.0	2.0
3	3	4	0.75	3.555555555555555
4	4	5	0.8	3.125 (Tartaglia)
5	75	94	0.797872340425532	3.141688888888889
6	679	851	0.797884841363102	3.141590444233810
7	2791	3498	0.797884505431675	3.141593089627162
8	3470	4349	0.797884571165785	3.141592571983821
9	16671	20894	0.797884560160812	3.141592658645841
10	53483	67031	0.797884560874819	3.141592653023172
11	123637	154956	0.797884560778544	3.141592653781322
12	177120	221987	0.797884560807615	3.141592653552392
13	654997	820917	0.797884560802127	3.141592653595605
14	832117	1042904	0.797884560803295	3.141592653586406
15	1487114	1863821	0.797884560802781	3.141592653590458
16	3806345	4770546	0.797884560802893	3.141592653589573
17	5293459	6634367	0.797884560802862	3.141592653589821
18	30273640	37942381	0.797884560802866	3.141592653589789
19	35567099	44576748	0.797884560802865	3.141592653589795

Here can be mentioned that the angle corresponding to x is $38.5858260136047\dots$ degrees, i.e. $x \approx \tan(38.5858260136047) = 0.797884560802865$. In practice, it is better to use the right triangle with sides of the suggested proportions. The triangle is symmetrical in its purpose; for a given circle (square) determines the square (circle). A very similar idea was proposed by a Russian engineer Edward Bing around the year 1877 [2-5].

References

1. Tartaglia N (1556) General trattato di numeri: et misvre [Link: https://goo.gl/VPIBgG](https://goo.gl/VPIBgG)
2. Szyszkowicz M (2016) Krótka historia ekierki Binga (in Polish: A short history of Bing's set square) East European Scientific J (EESJ) 12: 2.
3. Szyszkowicz M (2017). Ahmes' method to squaring the circle: European Journal of Mathematics and Computer Science (EJMCS), 4(1), 7-11.
4. Szyszkowicz M (2017) Ancient Egyptian Quadrature Executed Using A Set Square: Journal of Multidisciplinary Engineering Science and Technology (JMEST) 3(10): 5634-5636. [Link: https://goo.gl/zkw6oz](https://goo.gl/zkw6oz)
5. Bing E (1877) Der Kreiswinkel. Vermischtes: VDI-Z: Zeitschrift für die Entwicklung, Konstruktion, Produktion 21: 273-279.